Basic Trigonometric Identities

We have already seen the following identities:

Reciprocal Identities:

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent in Terms of Sine and Cosine:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

The Fundamental Identity:

$$\sin^2 x + \cos^2 x = 1$$

If we divide each term of the fundamental identity by $\sin^2 x$ or $\cos^2 x$, we can derive two more identities. These are called Pythagorean Identities because they are related to the Pythagorean Theorem:

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

Simplifying Expressions

We can use the identities above to simplify trigonometric expressions. One of the most common strategies is to start by rewriting the expression in terms of sines and/or cosines, then simplify from there.

Examples:

$$\frac{\tan x}{\sec x} \qquad \sin x + \cot x \cos x \qquad \frac{\tan x \csc x}{\sec x}$$

Writing One Function in Terms of Another

We sometimes want to write one identity in terms of another. For example, we could write the cosine function in terms of the sine by solving the fundamental identity to get $\cos x = \pm\sqrt{1-\sin^2 x}$.

Examples:

Write the tangent in terms of the sine.

Write the cotangent in terms of the cosine.

Using Identities to Find Function Values

If we know the value of one trigonometric function for an angle, we can use trigonometric identities to find the values of the other five functions.

Examples:

If $\tan\alpha = -2/3$ and α is in quadrant IV, find the values of the remaining five trigonometric functions.

Odd and Even Identities

An odd function is a function for which $f(-x) = -f(x)$, and an even function is one for which $f(-x) = f(x)$. The graph of an odd function is symmetric about the origin, and the graph of an even function is symmetric about the *y*-axis. Of the six trigonometric functions, the cosine and secant are even, and the others are odd.

Odd:	$\sin(-x) = -\sin x$	$\csc(-x) = -\csc x$
	$\tan(-x) = -\tan x$	$\cot(-x) = -\cot x$
Even:	$\cos(-x) = \cos x$	$\sec(-x) = \sec x$

Examples:

$\csc(-x)\tan(-x)$

$$\frac{1}{1+\cos(-x)} + \frac{1}{1-\cos x}$$